INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
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MARKSCHEME

May 2000

MATHEMATICAL METHODS

Standard Level

Paper 2

1. (a) Plan A: $1000, 1080, 1160\ldots$ Plan B: $1000, 1000(1.06), 1000(1.06)^2\ldots$

2nd month: \$ 1060, 3rd month: \$ 1123.60 ***(A1)(A1)***

[2 marks]

(b) For Plan A, $T_{12} = a + 11d$

$= 1000 + 11(80)$ ***(M1)***

$= \$\,1880$ ***(A1)***

For Plan B, $T_{12} = 1000(1.06)^{11}$ ***(M1)***

$= \$\,1898$ (to the nearest dollar) ***(A1)***

[4 marks]

(c) (i) For Plan A, $S_{12} = \frac{12}{2}[2000 + 11(80)]$ ***(M1)***

$= 6(2880)$

$= \$\,17\,280$ (to the nearest dollar) ***(A1)***

(ii) For Plan B, $S_{12} = \frac{1000(1.06^{12} - 1)}{1.06 - 1}$ ***(M1)***

$= \$\,16870$ (to the nearest dollar) ***(A1)***

[4 marks]

Total [10 marks]

2. (a) (i) $t = 0 \quad s = 800$

$t = 5 \quad s = 800 + 500 - 100 = 1200$ ***(M1)***

distance in first 5 seconds $= 1200 - 800$

$= 400\,\text{m}$ ***(A1)***

[2 marks]

(ii) $v = \frac{\mathrm{d}s}{\mathrm{d}t} = 100 - 8t$ ***(A1)***

At $t = 5$, velocity $= 100 - 40$ ***(M1)***

$= 60\,\text{m}\,\text{s}^{-1}$ ***(A1)***

[3 marks]

(iii) Velocity $= 36\,\text{m}\,\text{s}^{-1} \Rightarrow 100 - 8t = 36$ ***(M1)***

$t = 8$ seconds after touchdown. ***(A1)***

[2 marks]

(iv) When $t = 8$, $s = 800 + 100(8) - 4(8)^2$ ***(M1)***

$= 800 + 800 - 256$ ***(A1)***

$= 1344$ m ***(A1)***

[3 marks]

(b) If it touches down at P, it has $2000 - 1344 = 656$ m to stop. ***(M1)***

To come to rest, $100 - 8t = 0 \Rightarrow t = 12.5s$ ***(M1)***

Distance covered in 12.5 $s = 100(12.5) - 4(12.5)^2$ ***(M1)***

$= 1250 - 625$

$= 625$ ***(A1)***

Since 625 < 656, it can stop safely. ***(R1)***

[5 marks]

Total [15 marks]

3. (a) $\bar{x} = \$\,59$ ***(G2)***

OR

$$\bar{x} = \frac{10 \times 24 + 30 \times 16 + \ldots + 110 \times 10 + 130 \times 4}{24 + 16 + \ldots + 10 + 4}$$ ***(M1)***

$$= \frac{7860}{134}$$

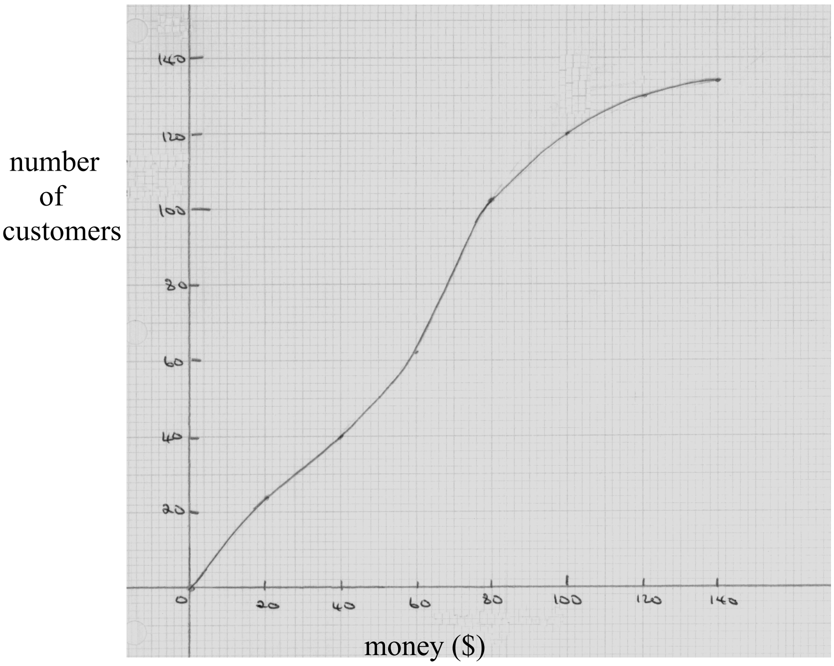
$$= \$\,59$$ ***(A1)***

[2 marks]

(b)

Money ($)	<20	<40	<60	<80	<100	<120	<140
Customers	24	40	**62**	**102**	**120**	**130**	**134**

(A1)



(A4)

Note: Award ***(A1)*** for the correct scale, ***(A1)*** for the points, and ***(A2)*** for the curve.

[5 marks]

(c) (i) $t = 2d^{2/3} + 3$

Mean $d = 59$ ***(M1)***

Mean $t \approx 2 \times (59)^{2/3} + 3$ ***(M1)***

≈ 33.3 min. (3 s.f.) (accept 33.2) ***(A1)***

(ii) $t > 37 \Rightarrow 2d^{2/3} + 3 > 37$ ***(M1)***

$2d^{2/3} > 34$

$d^{2/3} > 17$ ***(A1)***

$d > (17)^{3/2}$

$d > 70.1$

From the graph, when $d = 70.1, n = 82$ ***(A1)***

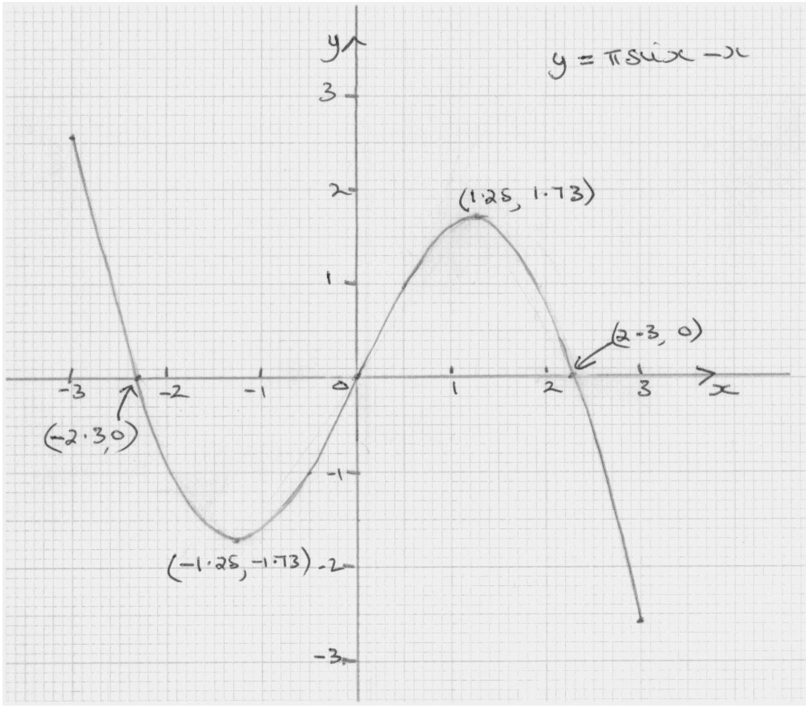
number of shoppers $= 134 - \mathbf{82}$ ***(A1)***

$= \mathbf{52}$ ***(A1)***

[8 marks]

Total [15 marks]

4. (a)



(A5)

Notes: Award ***(A1)*** for appropriate scales marked on the axes.
Award ***(A1)*** for the x-intercepts at $(\pm 2.3, 0)$.
Award ***(A1)*** for the maximum and minimum points at $(\pm 1.25, \pm 1.73)$.
Award ***(A1)*** for the end points at $(\pm 3, \mp 2.55)$.
Award ***(A1)*** for a smooth curve.
Allow some flexibility, especially in the middle three marks here.

[5 marks]

(b) $x = 2.31$ ***(A1)***

[1 mark]

(c) $\int (\pi \sin x - x)\,\mathrm{d}x = -\pi \cos x - \frac{x^2}{2} + C$ ***(A1)(A1)***

Note: Do not penalise for the absence of C.

Required area $= \int_0^1 (\pi \sin x - x)\,\mathrm{d}x$ ***(M1)***

$= 0.944$ ***(G1)***

OR

area $= 0.944$ ***(G2)***

[4 marks]

Total [10 marks]

5. (a) $\left|\begin{pmatrix}18\\24\end{pmatrix}\right| = 30\ \text{km}\,\text{h}^{-1}$ ***(A1)***

$\left|\begin{pmatrix}36\\-16\end{pmatrix}\right| = \sqrt{36^2 + (-16)^2}$

$= 39.4\ \text{km}\,\text{h}^{-1}$ ***(A1)***

[2 marks]

(b) (i) After ½ hour, position vectors are

$\begin{pmatrix}9\\12\end{pmatrix}$ and $\begin{pmatrix}18\\-8\end{pmatrix}$ ***(A1)(A1)***

(ii) At 6.30 a.m., vector joining their positions is

$\begin{pmatrix}9\\12\end{pmatrix} - \begin{pmatrix}18\\-8\end{pmatrix} = \begin{pmatrix}-9\\20\end{pmatrix}$ (or $\begin{pmatrix}9\\-20\end{pmatrix}$) ***(M1)***

$\left|\begin{pmatrix}-9\\20\end{pmatrix}\right|$ ***(M1)***

$= \sqrt{481}$ ($= 21.9$ km to 3 s.f.) ***(A1)***

[5 marks]

(c) The Toyundai must continue until its position vector is $\begin{pmatrix}18\\k\end{pmatrix}$ ***(M1)***

Clearly $k = 24$, *i.e.* position vector $\begin{pmatrix}18\\24\end{pmatrix}$ ***(A1)***

To reach this position, it must travel for 1 hour in total. ***(A1)***

Hence the crew starts work at 7.00 a.m. ***(A1)***

[4 marks]

(d) Southern (Chryssault) crew lays $800 \times 5 = 4000$ m ***(A1)***

Northern (Toyundai) crew lays $800 \times 4.5 = 3600$ m ***(A1)***

Total by 11.30 a.m. = 7.6 km

Their starting points were $24 - (-8) = 32$ km apart ***(A1)***

Hence they are now $32 - 7.6 = 24.4$ km apart ***(A1)***

[4 marks]

continued…

Question 5 continued

(e) Position vector of Northern crew at 11.30 a.m. is

$$\begin{pmatrix} 18 \\ 24 - 3.6 \end{pmatrix} = \begin{pmatrix} 18 \\ 20.4 \end{pmatrix}$$ ***(M1)(A1)***

Distance to base camp $= \left| \begin{pmatrix} 18 \\ 20.4 \end{pmatrix} \right|$ ***(A1)***

$= 27.2$ km

Time to cover this distance $= \dfrac{27.2}{30} \times 60$ ***(A1)***

$= 54.4$ minutes

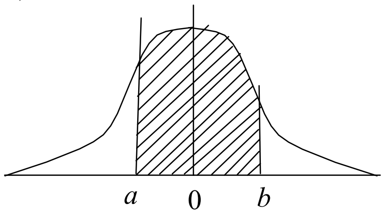
$= 54$ minutes (to the nearest minute) ***(A1)***

[5 marks]

Total [20 marks]

6. (i) (a) Let X be the lifespan in hours

$X \sim \mathrm{N}(57, 4.4^2)$



(i) $a = -0.455$ (3 s.f.) ***(A1)***

$b = 0.682$ (3 s.f.) ***(A1)***

(ii) (a) $\mathrm{P}(X > 55) = \mathrm{P}(Z > -0.455)$

$= 0.675$ ***(A1)***

(b) $\mathrm{P}(55 \le X \le 60) = \mathrm{P}\left(\frac{2}{4.4} \le Z \le \frac{3}{4.4}\right)$

$\approx \mathrm{P}(0.455 \le Z \le 0.682)$

$\approx 0.6754 + 0.752 - 1$ ***(A1)***

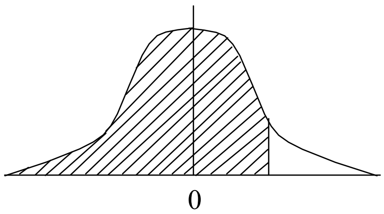
$= 0.428$ (3 s.f.) ***(A1)***

OR

$\mathrm{P}(55 \le X \le 60) = 0.428$ (3 s.f.) ***(G2)***

[5 marks]

(b) 90 % have died $\Rightarrow$ shaded area = 0.9 ***(M1)***



(A1)

Hence $t = 57 + (4.4 \times 1.282)$ ***(M1)***

$= 57 + 5.64$ ***(A1)***

$= 62.6$ hours ***(A1)***

OR

$t = 62.6$ hours ***(G3)***

[5 marks]

continued…

Question 6 continued

(ii) (a) 95 % between \$ 33000 and \$ 47000

$\Rightarrow 2.5\%$ above \$ 47000

$\Rightarrow \$\,47000 - \$\,40000$ corresponds to $z = 1.96$ ***(M1)***

$\Rightarrow \dfrac{7000}{\sigma} = 1.96$ ***(A1)***

$\Rightarrow \sigma = 3571$ ***(A1)***

$= \$\,3570$ (3 s.f.) ***(AG)***

[3 marks]

(b) $\text{SE} = \dfrac{\sigma}{\sqrt{50}}$ for samples of size 50

$= \dfrac{3571}{\sqrt{50}}$ ***(M1)***

$= \$\,505$ ***(A1)***

[2 marks]

(c) 95 % confidence interval for sample means

$= 40000 \pm 1.96\,\text{SE}$ ***(A1)***

$= 40000 \pm 1.96\,(505)$

$= 40000 \pm 990$ ***(A1)***

i.e. 95 % confidence that the mean lies between \$ 39010 and \$ 40990 ***(A1)***

[3 marks]

(d) Since $\$\,40900 < \$\,40990$, the mean of this sample lies within the 95 % confidence interval, *i.e.* there is not evidence that the salaries are higher in this province. ***(R1)*** ***(A1)***

[2 marks]

continued...

Question 6 continued

(iii) (a) Using a calculator:

L_1	14	15	15	12	11	18	
L_2	61	65	69	48	35	70	***(M2)***

Using an appropriate calculator function with correct arguments gives

$y = ax + b$ ***(A1)***

with $a = 5.027\ldots \quad = 5.03\ (3\ \text{s.f})$ ***(G1)***

$b = -13.216\ldots = -13.2\ (3\ \text{s.f})$ ***(G1)***

OR

$y = 5.03x - 13.2$ ***(G3)***

[5 marks]

(b) $y = ax + b$ with $x = 13$ gives ***(M1)***

$y = 13(5.027) - 13.216$ ***(A1)***

$= 52.13$

$= 52$ (to the nearest whole number) ***(A1)***

[3 marks]

(c) $r = 0.904\,69\ldots$ ***(G1)***

$= 0.905$ (3 s.f.) ***(G1)***

[2 marks]

Note: Some students may use the results $\Sigma y = na + b\Sigma x$, $\Sigma xy = a\Sigma x + b\Sigma x^2$, leading to $348 = 6a + 85b$, $5085 = 85a + 1235b$. Students attempting to find a, b using this method should be awarded generous marks, *e.g.* these 2 equations should be given 3 of the marks in part (a).

Total [30 marks]

7. (i) (a) $y = e^{2x}\cos x$

$\frac{dy}{dx} = e^{2x}(-\sin x) + \cos x(2e^{2x})$ ***(A1)(M1)***

$= e^{2x}(2\cos x - \sin x)$ ***(AG)***

[2 marks]

(b) $\frac{d^2y}{dx^2} = 2e^{2x}(2\cos x - \sin x) + e^{2x}(-2\sin x - \cos x)$ ***(A1)(A1)***

$= e^{2x}(4\cos x - 2\sin x - 2\sin x - \cos x)$ ***(A1)***

$= e^{2x}(3\cos x - 4\sin x)$ ***(A1)***

[4 marks]

(c) (i) At P, $\frac{d^2y}{dx^2} = 0$ ***(R1)***

$\Rightarrow 3\cos x = 4\sin x$ ***(M1)***

$\Rightarrow \tan x = \frac{3}{4}$

At P, $x = a$, *i.e.* $\tan a = \frac{3}{4}$ ***(A1)***

(ii) The gradient at any point $= e^{2x}(2\cos x - \sin x)$ ***(M1)***

Therefore, the gradient at P $= e^{2a}(2\cos a - \sin a)$

When $\tan a = \frac{3}{4}, \cos a = \frac{4}{5}, \sin a = \frac{3}{5}$ ***(A1)(A1)***

(by drawing a right triangle, or by calculator)

Therefore, the gradient at P$= e^{2a}\left(\frac{8}{5} - \frac{3}{5}\right)$ ***(A1)***

$= e^{2a}$ ***(A1)***

[8 marks]

(ii) (a) From the table, there are solutions in $[-3, -2], [1, 2]$ ***(A1)(A1)***

[2 marks]

(b) $2x^3 - 9x + 3 = 0$

$\Rightarrow 9x = 2x^3 + 3$ ***(A1)***

$x = \frac{2x^3 + 3}{9}$ ***(AG)***

and

$2x^3 = 9x - 3$

$x^3 = \frac{9x - 3}{2}$ ***(A1)***

$x = \sqrt[3]{\frac{9x - 3}{2}}$ ***(AG)***

[2 marks]

continued...

Question 7(ii) continued

(c) (i) $x = 0.342\,241$ ***(G1)***

Requires 6 iterations (accept any number from 5 to 9) ***(A1)***

[2 marks]

(ii) $g(x) = \dfrac{2x^3 + 3}{9}$

$g'(x) = \dfrac{6x^2}{9} = \dfrac{2x^2}{3}$ ***(M1)***

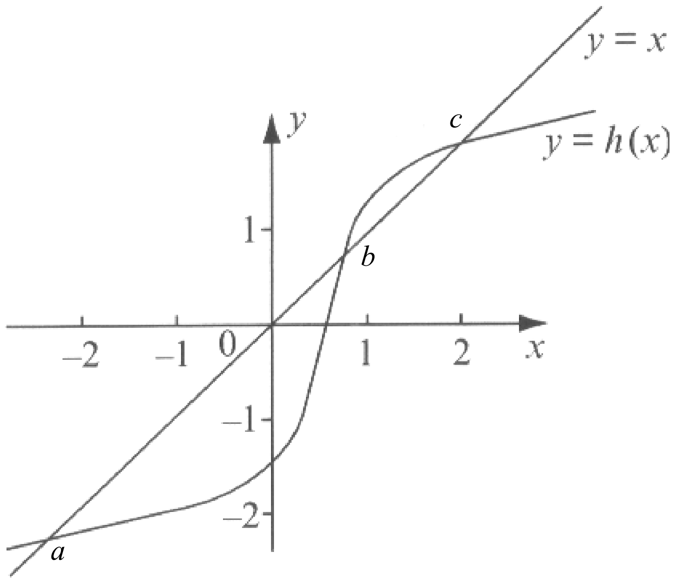
$g'(-3) = 6$ **and** $g'(-2) = \dfrac{8}{3}$ ***(M1)***

Since $|g'(x)| > 1$ in $[-3, -2]$, $g(x)$ will not converge in $[-3, -2]$. ***(R1)***

Similarly, $g'(1) = \dfrac{2}{3}$ **and** $g'(2) = \dfrac{8}{3}$ ***(M1)***

$g(x)$ will not converge, and if it does (near $x = 1$) it will give the same root as in part (i). ***(R1)***

[5 marks]



(iii) Gradient of $y = x$ is 1 ***(A1)***

It is clear from the graphs that at *a* and *c*, the gradient of $h(x)$ is less than 1, whereas at *b* it is greater than 1. ***(A1)***

Hence, $h(x)$ will give the solutions *a* and *c*, but not the solution at *b*. ***(R1)***

[3 marks]

(iv) Solutions: $-2.27163, 1.92939$ (6 s.f.) ***(G2)***

Note:	Award ***(G2)*** for 1 correct solution. Award ***(G1)*** for a correct solution that is not correct to **6** s.f.

[2 marks]

Total [30 marks]

8. (i) (a) $\boldsymbol{R} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ ***(A1)***

$\boldsymbol{S} = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$ ***(A1)***

$\boldsymbol{T} = \boldsymbol{RS}$

$= \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$ ***(M1)***

$= \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ ***(A1)***

[4 marks]

(b) Since $(0, 0)$ is invariant under ***T***, we need consider only one point on $y = 2x$ ***(M1)***

Take $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ for example.

$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ ***(M1)***

The line joining $(0, 0)$ and $(-1, 3)$ has equation $y = -3x$ ***(A1)***

[3 marks]

(ii) $\boldsymbol{Q}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - y \\ x + y \end{pmatrix}$

$\Rightarrow \boldsymbol{Q}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\boldsymbol{Q}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ ***(M1)(M1)***

$\Rightarrow \boldsymbol{Q} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ ***(A1)***

OR

Let $\boldsymbol{Q} = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

Then $\begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - y \\ x + y \end{pmatrix}$ ***(M1)***

$\Rightarrow ax + cy = x - y$

$bx + dy = x + y$ ***(M1)***

$\Rightarrow a = 1 \quad c = -1$

$b = 1 \quad d = 1$

$\boldsymbol{Q} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ ***(A1)***

[3 marks]

continued…

Question 8 continued

(iii) (a) Matrix is ***MN*** **(M1)**

$$= \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 0 & 3 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 7 \\ 1 & 8 \end{pmatrix}$$ **(A1)**

[2 marks]

(b) (i) $\begin{pmatrix} -1 & 7 \\ 1 & 8 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

(Or from definition of linear transformation) **(M1)**

[1 mark]

(ii) $\boldsymbol{a} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

$\boldsymbol{a}' = \begin{pmatrix} -1 & 7 \\ 1 & 8 \end{pmatrix} \begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} -34 \\ -26 \end{pmatrix}$ **(M1)**

A′ has coordinates $(-34, -26)$ **(A1)**

[2 marks]

(iii) $\begin{pmatrix} -1 & 7 \\ 1 & 8 \end{pmatrix} \boldsymbol{b} = \begin{pmatrix} -15 \\ 0 \end{pmatrix}$

$\Rightarrow \boldsymbol{b} = \begin{pmatrix} -1 & 7 \\ 1 & 8 \end{pmatrix}^{-1} \begin{pmatrix} -15 \\ 0 \end{pmatrix}$ **(M1)**

$= -\frac{1}{15} \begin{pmatrix} 8 & -7 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} -15 \\ 0 \end{pmatrix}$ **(A1)**

$= -\frac{1}{15} \begin{pmatrix} -120 \\ 15 \end{pmatrix} = \begin{pmatrix} 8 \\ -1 \end{pmatrix}$

B has coordinates $(8, -1)$ **(A1)**

OR

Solve $-x + 7y = -15$

$x + 8y = 0$ **(M1)**

$\Rightarrow x = 8, y = -1$ **(A1)**

B has coordinates $(8, -1)$ **(A1)**

[3 marks]

continued…

Question 8(iii)(b) continued

(iv) OABC is a parallelogram

$\Rightarrow \vec{OA} = -\vec{BC}$

$\boldsymbol{a} = \boldsymbol{b} - \boldsymbol{c}$

$\boldsymbol{c} = \boldsymbol{b} - \boldsymbol{a}$ ***(M1)***

$= \begin{pmatrix} 8 \\ -1 \end{pmatrix} - \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ***(A1)***

$\boldsymbol{c}' = \begin{pmatrix} -1 & 7 \\ 1 & 8 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 19 \\ 26 \end{pmatrix}$ ***(A1)***

[3 marks]

(v) $\boldsymbol{a} \cdot \boldsymbol{c} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= 0$

$\Rightarrow$ **a** perpendicular to **c** ***(M1)***

$\Rightarrow$ OABC is a rectangle ***(R1)***

$\boldsymbol{a}' \cdot \boldsymbol{c}' = \begin{pmatrix} -34 \\ -26 \end{pmatrix} \begin{pmatrix} 19 \\ 26 \end{pmatrix}$

$\neq 0$ ***(M1)***

$\Rightarrow$ O′A′B′C′ is not a rectangle ***(R1)***

[4 marks]

(vi) Area OABC $= |\boldsymbol{a}||\boldsymbol{c}|$ since it is a rectangle ***(M1)***

$= \sqrt{52}\sqrt{13}$ ***(A1)***

$= 26$ sq units ***(A1)***

Area O′A′B′C′ $= \left| \det \begin{pmatrix} -1 & 7 \\ 1 & 8 \end{pmatrix} \right| \times 26$ ***(M1)***

$= 15 \times 26$

$= 390$ sq units ***(A1)***

[5 marks]

Total [30 marks]